Enrollment No: ______________________ Exam Seat No: ______________________

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name : Engineering Mathematics - I

Subject Code : 4TE01EMT2 **Branch: B. Tech (All)**

Semester : 1 **Date : 14/03/2019** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) If $y=\sin 2x\cos 2x$ then y_n equal to

(A) $\frac{1}{2}(4)^n\cos\left(\frac{n\pi}{2}+4x\right)$ (B) $\frac{1}{2}(4)^n\sin\left(\frac{n\pi}{2}+4x\right)$

(C) $\frac{1}{2}(4)^n\sin\left(\frac{n\pi}{2}+2x\right)$ (D) none of these

b) nth derivative of $y=\frac{1}{x+a}$ is

(A) $\frac{(-1)^n n!}{(x+a)^{n+1}}$ (B) $\frac{(-1)^{n-1} n!}{(x+a)^{n+1}}$ (C) $\frac{(-1)^n n!}{(x+a)^{n}}$ (D) none of these

c) If $1+y=e^{-x}$ or $y=e^{-x}-1$, then x equal to

(A) $-y+\frac{y^2}{2}-\frac{y^3}{3}+\frac{y^4}{4}+.....$ (B) $y-\frac{y^2}{2}+\frac{y^3}{3}-\frac{y^4}{4}+.....$

(C) $y+\frac{y^2}{2}+\frac{y^3}{3}+\frac{y^4}{4}+.....$ (D) None of these

d) The series $x+\frac{x^3}{3!}+\frac{x^5}{5!}+.....$ represent expansion of

(A) $\sin x$ (B) $\cos x$ (C) $\sinh x$ (D) $\cosh x$

e) $\lim_{x\to\infty} x^n e^{-ax}$ (n being a positive integer and $a>0$) = _____

(A) – 1 (B) 0 (C) 1 (D) None of these

f) $\lim_{x\to\infty} x\left(a^{\frac{1}{x}}-1\right)=$ _____

(A) $\log_e a$ (B) 0 (C) 1 (D) none of these

g) If $\frac{\partial(u,v)}{\partial(x,y)}\times\frac{\partial(x,y)}{\partial(u,v)}$ is equal to

(A) 1 (B) – 1 (C) zero (D) none of these

h) If $u = f\left(\frac{x}{y}\right)$ then

(A) $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = 0$ (B) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$ (C) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = u$ (D) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$

i) If $Q = r\cot\theta$, then $\frac{\partial Q}{\partial r}$ is equal to

(A) $\cot\theta$ (B) $-\cos ec^2\theta$ (C) $\cot\theta - r\cos ec^2\theta$ (D) $\frac{1}{2}\cot\theta$

j) If $u = y^x$, then $\frac{\partial u}{\partial x}$ is

(A) xy^{x-1} (B) 0 (C) $y^x \log x$ (D) none of these

k) If 1, α_1, α_2, ..., α_{n-1} are the n roots of unity, then $(1+\alpha_1)(1+\alpha_2).....(1+\alpha_{n-1})$ is equal to

(A) $n-1$ (B) n (C) – 1 (D) none of these

l) If $\left(\frac{1-i}{1+i}\right)^{100} = a + ib$, then

(A) $a = 2,\ b = -1$ (B) $a = 1,\ b = 0$ (C) $a = 0,\ b = 1$ (D) $a = -1,\ b = 2$

m) An $n \times n$ homogeneous system of equations $AX = 0$ is given. The rank of A is $r < n$. Then the system has

(A) $n - r$ independent solutions (B) r independent solutions (C) no solution (D) n independent solutions

n) The rank of the diagonal matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is

(A) 1 (B) 2 (C) 3 (D) – 2

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions (14)

a) If $y = \frac{x^4}{(x-1)(x-2)}$ then find y_n. **(5)**

b) Expand $e^{\sin x}$ as a series of ascending power of x upto x^4. **(5)**

c) If $u = \log\left(x^3 + y^3 - x^2y - xy^2\right)$ then prove that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right)^2 u = -\frac{4}{(x+y)^2}$ **(4)**

Q-3 Attempt all questions (14)

a) If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$ then prove that **(5)**

$x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0$.

b) Prove that $(1+x)^x = 1+x^2-\frac{1}{2}x^3+\frac{5}{6}x^4-.....$ **(5)**

c) Evaluate: $\lim_{x\to\frac{\pi}{4}}(1-\tan x)\sec 2x$ **(4)**

Q-4 **Attempt all questions** **(14)**

a) Evaluate: $\lim_{x\to 0}\left(\frac{1}{x}\right)^{1-\cos x}$ **(5)**

b) If $x = r\sin\theta\cos\phi,\ y = r\sin\theta\sin\phi,\ z = r\cos\theta$ then find $\frac{\partial(x,\ y,\ z)}{\partial(r,\ \theta,\ \phi)}$. **(5)**

c) Expand $\log x$ in powers of $(x-2)$. **(4)**

Q-5 **Attempt all questions** **(14)**

a) If $u = \sec^{-1}\left(\frac{x^2+y^2}{x-y}\right)$ then prove that **(5)**

$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2} = -\cot u\left(\cot^2 u+2\right)$.

b) Evaluate: $\lim_{x\to 0}\frac{e^x+e^{-x}-x^2-2}{\sin^2 x-x^2}$ **(5)**

c) If $y = \cos x\cos 2x\cos 3x$ then find y_n . **(4)**

Q-6 **Attempt all questions** **(14)**

a) The power consumed in an electric resistor is given by $P = \frac{E^2}{R}$ (in watts). If $E = 200$ volts and $R = 8$ ohms, by how much does the power change if E is decreased by 5 volts and R is decreased by 0.20 ohms? **(5)**

b) Prove that $\left(\frac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta}\right)^n = \cos\left(\frac{n\pi}{2}-n\theta\right)+i\sin\left(\frac{n\pi}{2}-n\theta\right)$ **(5)**

c) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix}1 & -2\\ -5 & 4\end{bmatrix}$. **(4)**

Q-7 **Attempt all questions** **(14)**

a) Reduce the matrix $A = \begin{bmatrix}2 & 3 & 4 & 5\\ 3 & 4 & 5 & 6\\ 4 & 5 & 6 & 7\\ 9 & 10 & 11 & 12\end{bmatrix}$ to the normal form and find its rank. **(5)**

b) Find the continued product of all the values of $\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)^{\frac{3}{4}}$. **(5)**

c) Prove that $\sec h^{-1}(\sin\theta) = \log\cot\frac{\theta}{2}$. **(4)**

Q-8 **Attempt all questions** **(14)**

a) Examine whether the following equations are consistent and solve them if they are consistent. **(5)**

$2x+6y+11=0,\ 6x+20y-6z+3=0,\ 6y-18z+1=0$

b) Find the fourth roots of unity and sketch them on the unit circle. **(5)**

c) Verify Caley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. **(4)**